EECE 340 Quiz 2
SUMMER 2011

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PROBLEM 1

a) $$y(t) = \int_{-\infty}^{t} e^{-4(t-\lambda)} x(\lambda-1)\, d\lambda$$

This can be thought of as a convolution of the input with a unit step-like impulse response. That is

$$y(t) = \int_{-\infty}^{t} e^{-4(t-1-\lambda+1)} \cdot u(t-\lambda)\, x(\lambda-1)\, d\lambda$$

$$= \int_{-\infty}^{t} e^{4} \cdot e^{-4(t-(\lambda-1))} u(t-1-(\lambda-1))\, x(\lambda-1)\, d\lambda$$

a change of variable $\lambda' = \lambda - 1$ gives

$$y(t) = \int_{-\infty}^{t-1} e^{4} \cdot e^{-4(t-\lambda')} u(t-1-\lambda')\, x(\lambda')\, d\lambda'$$

$$= \int_{-\infty}^{t-1} e^{-4(t-1-\lambda')} u(t-1-\lambda')\, x(\lambda')\, d\lambda'$$

which is the same as

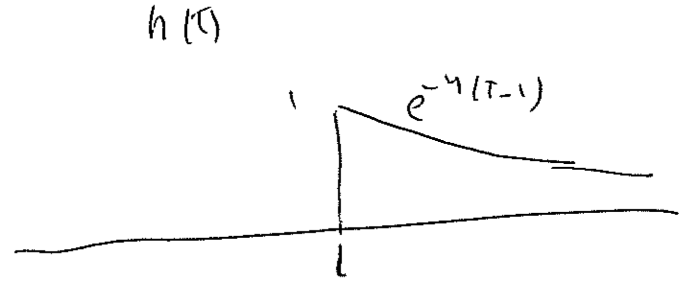
$$y(t) = h(t) * x(t) \quad \text{where}$$

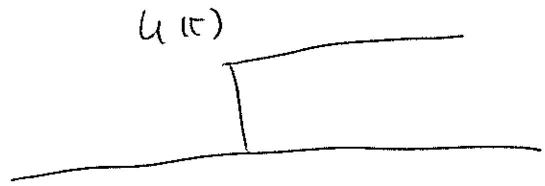
$$h(t) = e^{-4(t-1)} u(t-1)$$

- since $h(t) = 0 \;\forall t < 0$ system is causal
- since $\int |h(t)|\, dt < \infty$ or $H(s)$ has pole in LHP the system is stable.

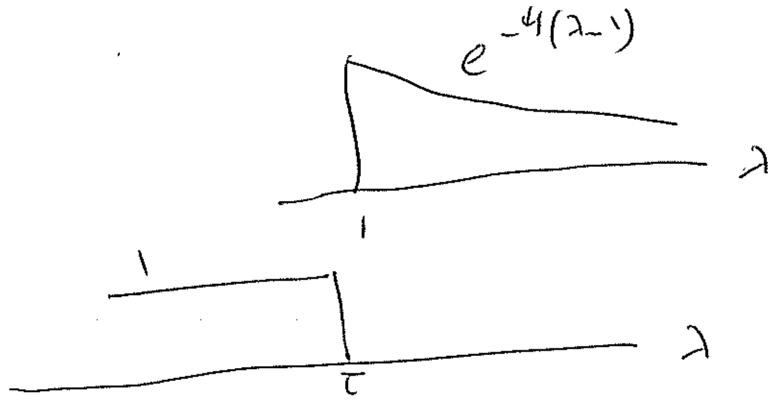
- if input is $u(t)$

Then $y(t) = h(t) * u(t)$





graphical convolution gives

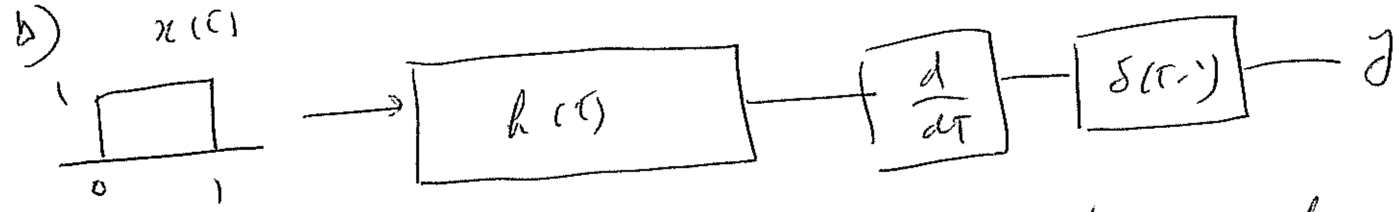


for $t<1$ $\quad y(t)=0 \Rightarrow y(\tfrac{1}{2})=0$

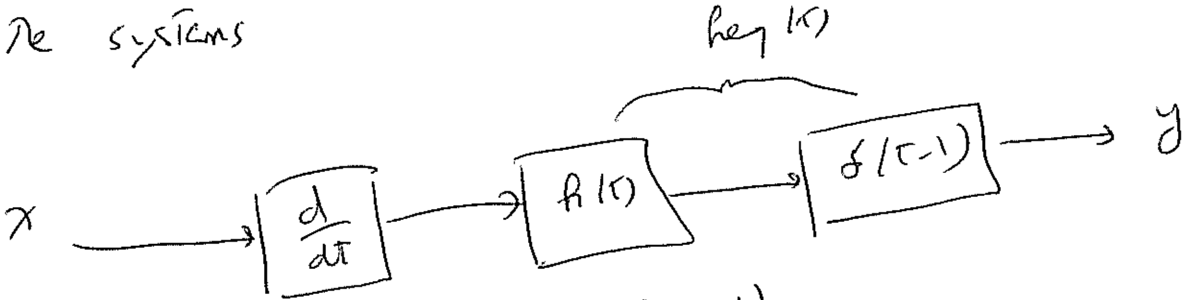
(or can be seen from the integral:

$$y(\tfrac{1}{2}) = \int_{-\infty}^{\frac{1}{2}} e^{-4(\frac{1}{2}-\lambda)} \overbrace{u(-\tfrac{1}{2})}^{=0} \, d\lambda = 0 \Big)$$

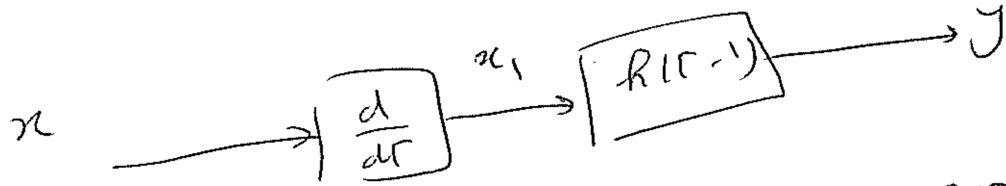
b)



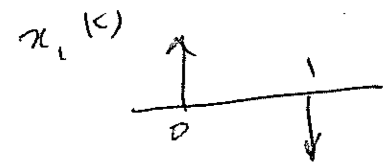
since $h(t)$ & $\frac{d}{dt}$ are LTI, can interchange order of the systems



also $h_{eq}(t) = h(t) * \delta(t-1) = h(t-1)$



for the given input $x_1(t) = \delta(t) - \delta(t-1)$

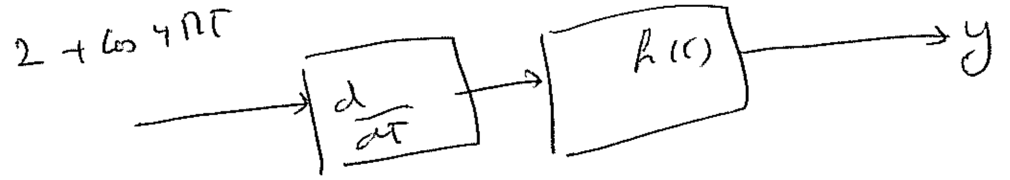


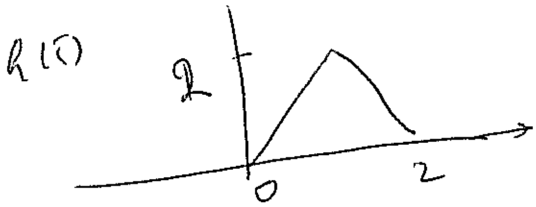
$$\therefore \quad y(t) = h(t-1) - h(t-2)$$

$$= e^{-(t-1)} \cos 3(t-1)\, u(t-1) + \frac{e^{-(t-2)}}{t-1} u(t-2)$$

$$- e^{-(t-2)} \cos 3(t-2)\, u(t-2) + \frac{e^{-(t-3)}}{t-2} u(t-3)$$

c)



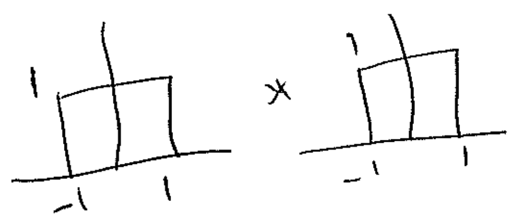


to find energy in output, use Parsevals Theorem

$$Y(\omega) = j\omega \cdot H(\omega) \cdot X(\omega)$$

$$X(\omega) = 4\pi\delta(\omega) + \pi[\delta(\omega - 4\pi) + \delta(\omega + 4\pi)]$$

Also if $h_1(t) = h_2(t) * h_2(t)$ where $h_2(t)$



$$\therefore H_1(\omega) = (H_2(\omega))^2$$

$$= \left(\frac{2\sin\omega}{\omega}\right)^2$$

$$h(t) = h_1(t-1) \Rightarrow H(\omega) = e^{j\omega}\, 4\left(\frac{\sin\omega}{\omega}\right)^2$$

$$Y(\omega) = j\omega e^{j\omega}\, 4\left(\frac{\sin\omega}{\omega}\right)^2 \cdot [4\pi\delta(\omega) + \pi\delta(\omega - 4\pi) + \pi\delta(\omega + 4\pi)]$$

$$= j0\, e^{j0}\, 4\,\mathrm{sinc}^2(0) \cdot 4\pi\,\delta(\omega)$$

$$+ j4\pi e^{j4\pi}\, 4\left(\frac{\sin 4\pi}{4\pi}\right)^2 \cdot \pi \cdot \delta(\omega - 4\pi)$$

$$+ j4\pi e^{-j4\pi}\, 4\left(\frac{\sin(-4\pi)}{-4\pi}\right)^2 \cdot \pi\,\delta(\omega + 4\pi) = 0\,!$$

$$\therefore \boxed{Y(\omega) = 0} \rightarrow E_y = 0$$

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PROBLEM 2

$$H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n};$$

a) for $h(n)$ causal, $h(n) \equiv 0 \ \forall n < 0 \Rightarrow H(z) = h(0) + h(1) z^{-1} + h_2 (z^{-2}) + \dots$

$\Rightarrow \lim_{z \to \infty} H(z)$ is $h(0)$ which is finite

$\therefore$ CAUSAL systems will have $\lim_{z \to \infty} H(z) < \infty$

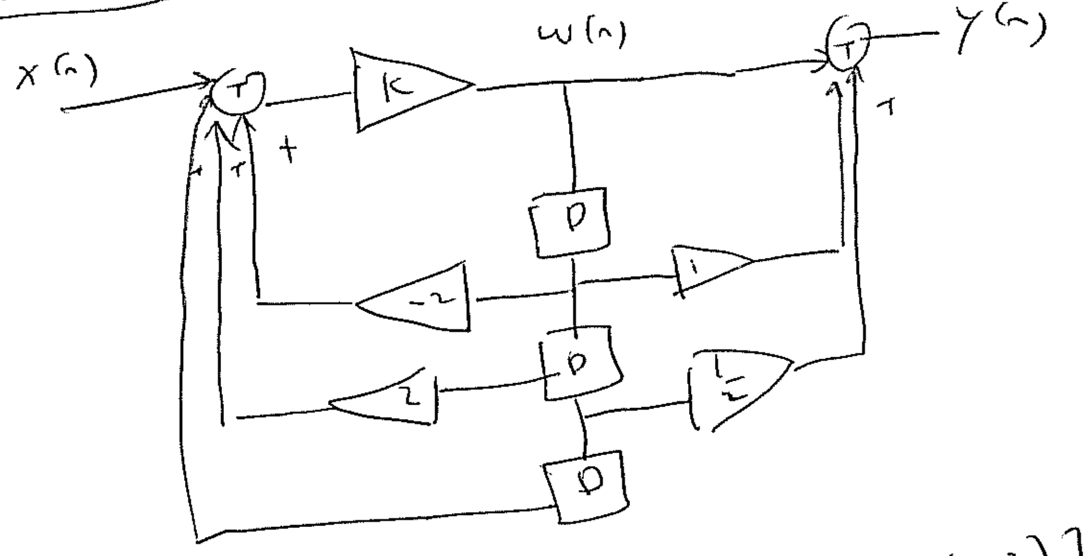
b) i- $$H(z) = \frac{(z - \frac{1}{3})^4}{(z - \frac{1}{2})^3} = z \frac{(1 - \frac{1}{3} z^{-1})^4}{(1 - \frac{1}{2} z^{-1})^3}$$

for this system to be stable, R.O.C must include the unit circle $\Rightarrow$ $|z| > \frac{1}{2}$ is the corresponding ROC.

ii- $\lim_{z \to \infty} H(z) \Rightarrow \infty$ $\Rightarrow$ it cannot be causal

which can also be seen by the z term multiplying the numerator

PROBLEM 3



a) $w(n) = K\left[x(n) - 2w(n-1) + 2w(n-2) + w(n-3)\right]$

$y(n) = w(n) + w(n-1) + \frac{1}{2}w(n-2)$

Take z-xform, you obtain

$W(z) = K\left[X(z) - 2z^{-1}W(z) + 2z^{-2}W(z) + z^{-3}W(z)\right]$

$\Rightarrow W(z)\left(\frac{1}{K} + 2z^{-1} - 2z^{-2} - z^{-3}\right) = X(z)$

Also $Y(z) = \left(1 + z^{-1} + \frac{1}{2}z^{-2}\right)W(z)$

$$\Rightarrow \frac{Y(z)}{X(z)} = \frac{1 + z^{-1} + \frac{1}{2}z^{-2}}{\frac{1}{K} + 2z^{-1} - 2z^{-2} - z^{-3}} = H(z)$$

b) $H(z)$ has a pole at $z = -\frac{1}{2}$

$$\Rightarrow H(z) = \frac{N(z)}{(1+\frac{1}{2}z^{-1})(D(z))}$$

$\Rightarrow$ $\frac{1}{K} + 2z^{-1} - 2z^{-2} - z^{-3}$ has a factor of $(1+\frac{1}{2}z^{-1})$ in it

$\Rightarrow$ dividing this poly'l by $1+\frac{1}{2}z^{-1}$ should have zero remainder

(This will also give the remainder poly'l)

(or $H(z) \to \infty$ at $z = -\frac{1}{2}$ to find K)

$$\begin{array}{r|l} & -2z^{-2}+4 \\ \hline 1+\frac{1}{2}z^{-1} & -z^{-3}-2z^{-2}+2z^{-1}+\frac{1}{K} \\ & \underline{-z^{-3}-2z^{-2}} \\ & 2z^{-1}+\frac{1}{K} \\ & \underline{2z^{-1}+4} \\ & \frac{1}{K}-4 \end{array}$$

$\Rightarrow \frac{1}{K} - 4 = 0$

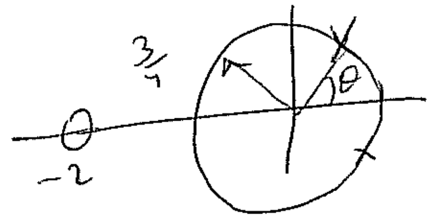
or $\boxed{K = 4}$

c)
$$H(z) = \frac{1+z^{-1}+\frac{1}{2}z^{-2}}{4+2z^{-1}-2z^{-2}-z^{-3}}$$

$$= \frac{1+z^{-1}+\frac{1}{2}z^{-2}}{4(1+\frac{1}{2}z^{-1})(1-\frac{1}{2}z^{-2})} = \frac{N(s)}{4(1+\frac{1}{2}z^{-1})(1-\frac{1}{\sqrt{2}}z^{-1})(1+\frac{1}{\sqrt{2}}z^{-1})}$$

since system is causal & all poles are inside unit circle $\Rightarrow$ system is stable.

PROBLEM 4

a) 

$\theta = \frac{\pi}{3}$

2 poles at $\frac{3}{4}e^{\pm j\frac{\pi}{3}}$

Two zeros at $z = -2$
$z = 0$

$$\therefore \quad H(z) = \frac{(z+2)z}{(z - \frac{3}{4}e^{-j\frac{\pi}{3}})(z - \frac{3}{4}e^{+j\frac{\pi}{3}})}$$

Now $h_1(n) = (-1)^n h(n) = (a)^n h(n)$

$$\Rightarrow \quad H_1(z) = H(a^{-1}z) = H(-z)$$

$$H_1(z) = \frac{(-z+2)(-z)}{(-z - \frac{3}{4}e^{-j\frac{\pi}{3}})(-z - \frac{3}{4}e^{j\frac{\pi}{3}})} = \frac{(2-z)(-z)}{(z + \frac{3}{4}e^{-j\frac{\pi}{3}})(z + \frac{3}{4}e^{j\frac{\pi}{3}})}$$

b) $$H_2(z) = \frac{1}{H_1(z)} = \frac{(z + \frac{3}{4}e^{-j\frac{\pi}{3}})(z + \frac{3}{4}e^{j\frac{\pi}{3}})}{(2-z)(-z)}$$

$$= \frac{-z(1 + \frac{3}{4}e^{-j\frac{\pi}{3}}z^{-1})(1 + \frac{3}{4}e^{j\frac{\pi}{3}}z^{-1})}{(1 - 2z^{-1})(-z)}$$

for this recovery filter to be stable
it is not causal (pole outside unit circle)

$$H_2(z) = \frac{(1 + \frac{3}{4}e^{-j\frac{\pi}{3}}z^{-1})(1 + \frac{3}{4}e^{-j\frac{\pi}{3}}z^{-1})}{1 - 2z^{-1}}$$

$$H_2(z) = \frac{1 + \left(\frac{3}{4} e^{i\frac{\pi}{3}} + \frac{3}{4} e^{-i\frac{\pi}{3}}\right) z^{-1} + \frac{9}{16} z^{-2}}{1 - 2z^{-1}}$$

$$= \frac{1 + \frac{3}{4} \cos\frac{\pi}{3} z^{-1} + \frac{9}{16} z^{-2}}{1 - 2z^{-1}} = \frac{1 + \frac{3}{8} z^{-1} + \frac{9}{16} z^{-2}}{1 - 2z^{-1}}$$

which is not implematable (noncausal)

d) for the system to be implematable

$H_1(z) = H\left(\frac{z}{a}\right)$ $\quad \left(h_1[n] = (a)^n h(n) \right)$

should have ~~poles~~ zeros inside the unit circle

In otherwords,

$$H_0(z) = \frac{1 + 2z^{-1}}{\left(1 - \frac{3}{4} e^{-i\frac{\pi}{3}} z^{-1}\right)\left(1 - \frac{3}{4} e^{i\frac{\pi}{3}} z^{-1}\right)}$$

should produce an $H_1(z)$ with zero inside unit circle

$$H_1(z) = H\left(\frac{z}{a}\right) = \frac{1 + 2\left(\frac{z}{a}\right)^{-1}}{\left[1 - \frac{3}{4} e^{-i\frac{\pi}{3}} \left(\frac{z}{a}\right)^{-1}\right] \left[1 - \frac{3}{4} e^{i\frac{\pi}{3}} \left(\frac{z}{a}\right)^{-1}\right]}$$

$$= \frac{1 + 2az^{-1}}{\left(1 - \frac{3}{4} a e^{-i\frac{\pi}{3}} z^{-1}\right)\left(1 - \frac{3}{4} a e^{i\frac{\pi}{3}} z^{-1}\right)}$$

need $2a < 1$ $\quad$ & $\underline{\frac{3}{4} a < 1}$

$2a < 1$ → zero inside circle

$\frac{3}{4} a < 1$ → $H_1(z)$ should still be stable

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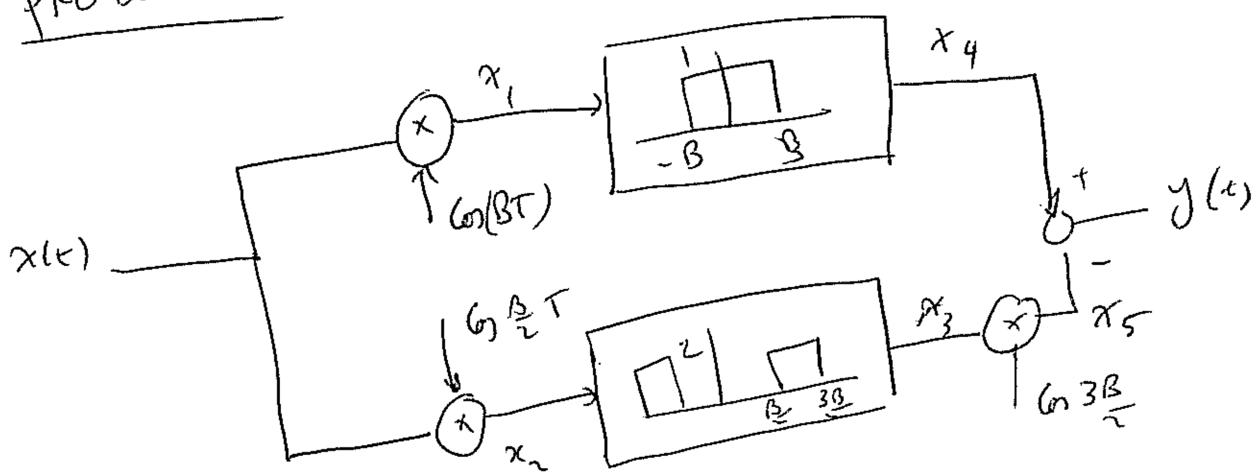
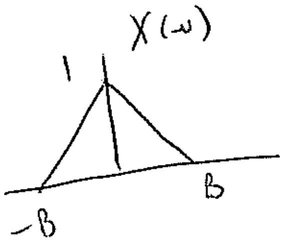
$\therefore\ a < \frac{1}{2}$ & $a < \frac{4}{3}$ $\Rightarrow$ choose any $a < \frac{1}{2}$ will work.

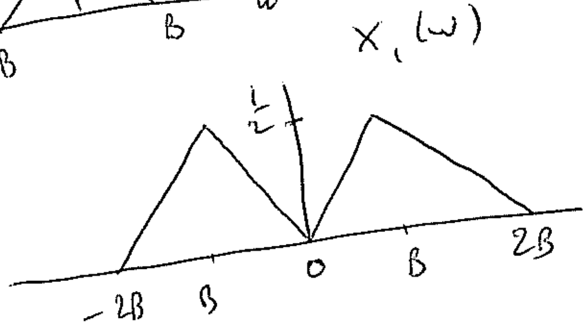
In this case,

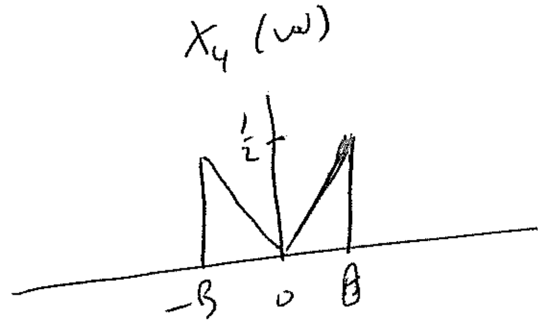
$$H_2(z) = \frac{1 + \frac{3a}{8} z^{-1} + \frac{9a^2}{16} z^{-2}}{1 - 2a z^{-1}}$$ is implementable as a causal stable filter.

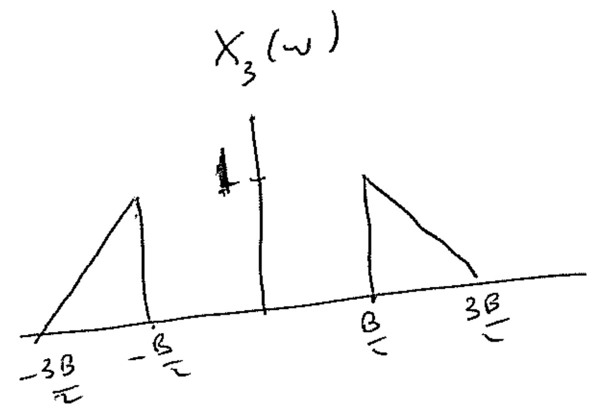
PROBLEM 5

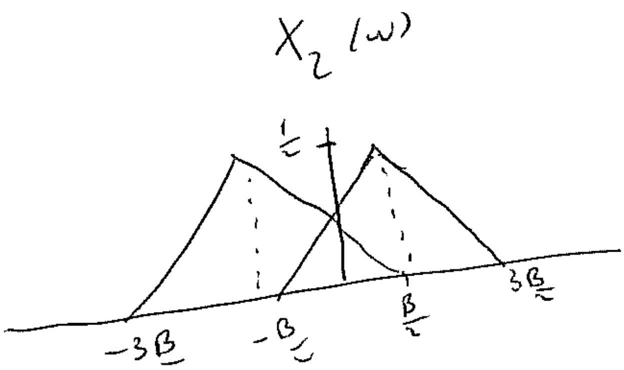




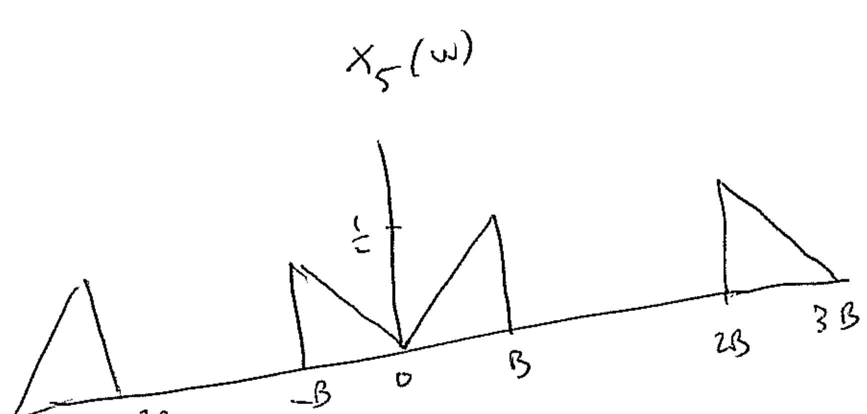




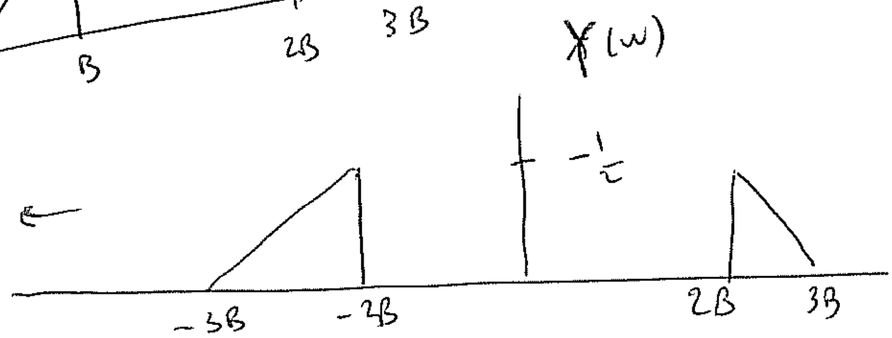




$$Y(\omega) = X_4(\omega) - X_5(\omega)$$



SSB
AM



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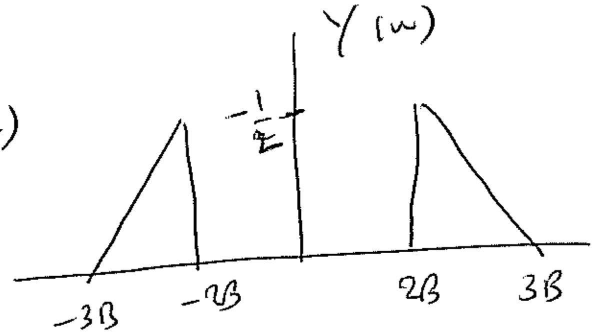
c)



x(t) can be recovered using the following demodulation.